

AP CALCULUS AB

PRACTICE EXAM

SECTION I
MULTIPLE-CHOICE
QUESTIONS
2008

1. $\int \cos(3x) dx =$

(A) $-3\sin(3x) + C$

(B) $-\frac{1}{3}\sin(3x) + C$

C (C) $\frac{1}{3}\sin(3x) + C$

(D) $\sin(3x) + C$

(E) $3\sin(3x) + C$

$u = 3x$
 $\frac{1}{3} du = dx$
 $\frac{1}{3} \int \cos u du$
 $\frac{1}{3} \sin(3x) + C$

2. $\lim_{x \rightarrow 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is $= \lim_{x \rightarrow 0} \frac{x^3(2x^3 + 6)}{x^3(4x^2 + 3)} = \frac{2(0^3) + 6}{4(0^2) + 3} = \frac{6}{3} = 2$

D (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \leq 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$

3. The function f is defined above. For what value of k , if any, is f continuous at $x = 2$?

C (A) 1

(B) 2

(C) 3

(D) 7

(E) No value of k will make f continuous at $x = 2$.

$x^2 - 3x + 9 = kx + 1 \quad x = 2$

$2^2 - 3(2) + 9 = k(2) + 1$

$7 = 2k + 1$

$6 = 2k$

$3 = k$

4. If $f(x) = \cos^3(4x)$, then $f'(x) = 3(\cos 4x)^2(-\sin 4x)(4)$

B (A) $3\cos^2(4x)$

$-12\cos^2 4x \sin 4x$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

5. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

C (A) -1

(B) 0

(C) 2

(D) $\frac{3 - \sqrt{105}}{4}$

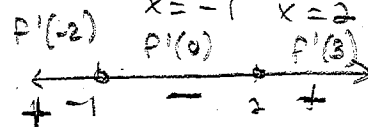
(E) $\frac{3 + \sqrt{105}}{4}$

$f'(x) = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1 \quad x = 2$



6. Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

B

- (A) $y = 21x + 2$
 (B) $y = 21x - 19$
 (C) $y = 11x - 9$
 (D) $y = 10x + 2$
 (E) $y = 10x - 8$

$$f'(x) = (2x-1)^5(1) + (x+1)(5)(2x-1)^4(2)$$

$$f'(x) = (2x-1)^4 [2x-1 + 10(x+1)]$$

$$f'(x) = (2x-1)^4 (12x+9)$$

$$m = f'(1) = 21 \quad \text{Point } (1, 2)$$

$$y - 2 = 21(x - 1)$$

$$y = 21x - 19$$

7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$

A

- (A) $2e^{\sqrt{x}} + C$
 (B) $\frac{1}{2}e^{\sqrt{x}} + C$
 (C) $e^{\sqrt{x}} + C$
 (D) $2\sqrt{x}e^{\sqrt{x}} + C$
 (E) $\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$

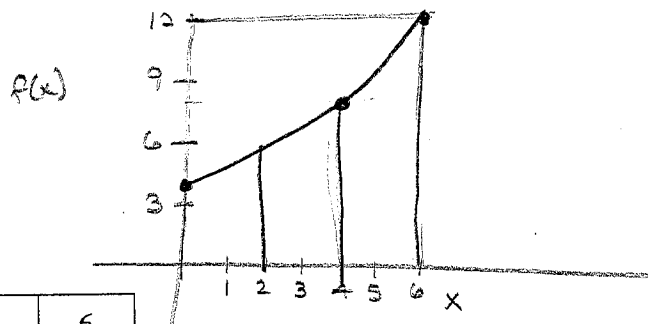
$$u = x^{1/2}$$

$$du = \frac{1}{2}x^{-1/2} dx$$

$$2du = \frac{1}{\sqrt{x}} dx$$

$$2 \int e^u du$$

$$2e^{\sqrt{x}} + C$$



x	0	2	4	6
$f(x)$	4	k	8	12

8. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above.

D The trapezoidal approximation for $\int_0^6 f(x) dx$ found with 3 subintervals of equal length is 52. What

is the value of k ?

- (A) 2 (B) 6 (C) 7 (D) 10 (E) 14

$$52 = \frac{1}{2}(2)(4+k) + \frac{1}{2}(2)(k+8) + \frac{1}{2}(2)(8+12)$$

$$52 = 4+k+k+8+20$$

$$20 = 2k$$

$$k = 10$$

9. A particle moves along the x -axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

C

- (A) -10 (B) -5 (C) -3 (D) 3 (E) 17

$$v(t) = 4 - 6t^2$$

$$x(t) = \int (4 - 6t^2) dt$$

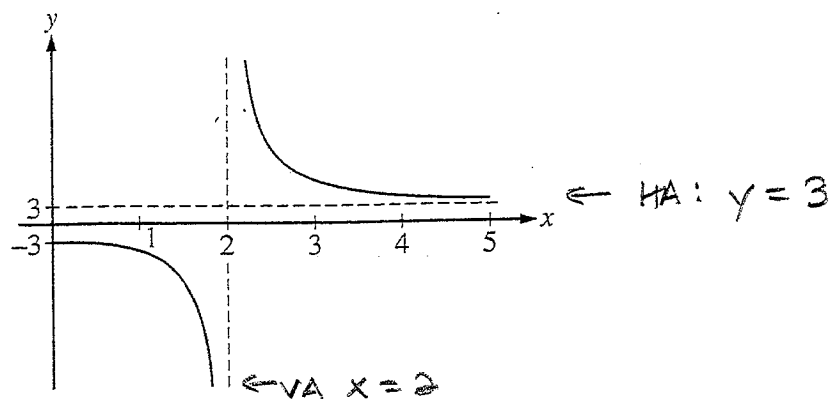
$$x(t) = 4t - 2t^3 + C$$

$$(1, 7) \quad 7 = 4(1) - 2(1^3) + C$$

$$5 = C$$

$$x(t) = 4t - 2t^3 + 5$$

$$x(2) = 8 - 16 + 5 = -3$$



10. The function f is given by $f(x) = \frac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of f . Which of the following could be the values of the constants a and b ?

(A) $a = -3, b = 2$

(B) $a = 2, b = -3$

(C) $a = 2, b = -2$

(D) $a = 3, b = -4$

(E) $a = 3, b = 4$

$$f(x) = \frac{ax^2 + 12}{x^2 + b} \Rightarrow HA: \frac{ax^2}{x^2} = 3$$

$$\boxed{a = 3}$$

$$x^2 - 4 = x^2 + b$$

$$b = -4$$

11. What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\frac{1}{e}$

(B) $-\frac{3}{4e}$

(C) $-\frac{1}{4e}$

(D) $\frac{1}{4e}$

(E) $\frac{1}{e}$

$$y' = \frac{(x+1)(-e^{-x}) - (e^{-x})(1)}{(x+1)^2}$$

$$y' = \frac{-e^{-x}(x+1+1)}{(x+1)^2} = \frac{-(x+2)}{e^x(x+1)^2}$$

$$m = \frac{-(1+2)}{e^1(1+1)^2} = \frac{-3}{4e}$$

12. If $f'(x) = \frac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2

(B) $\ln 25$

(C) $5 + \frac{2}{e} - \frac{2}{e^2}$

(D) 6

(E) 25

$$f(x) = 2 \ln|x| + C$$

$$2 + 4 = 6$$

$$f(x) = \int \frac{2}{x} dx = 2 \ln|x| + C$$

$$f(x) = 2 \ln|x| + C$$

$$5 = 2 \ln \sqrt{e} + C$$

$$5 = 2 \ln e^{1/2} + C$$

$$5 = 2 \cdot \frac{1}{2} \ln e + C$$

$$5 = 1 + C$$

$$4 = C$$

13. $\int (x^3 + 1)^2 dx =$

(A) $\frac{1}{7}x^7 + x + C$

(B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

(C) $6x^2(x^3 + 1) + C$

(D) $\frac{1}{3}(x^3 + 1)^3 + C$

(E) $\frac{(x^3 + 1)^3}{9x^2} + C$

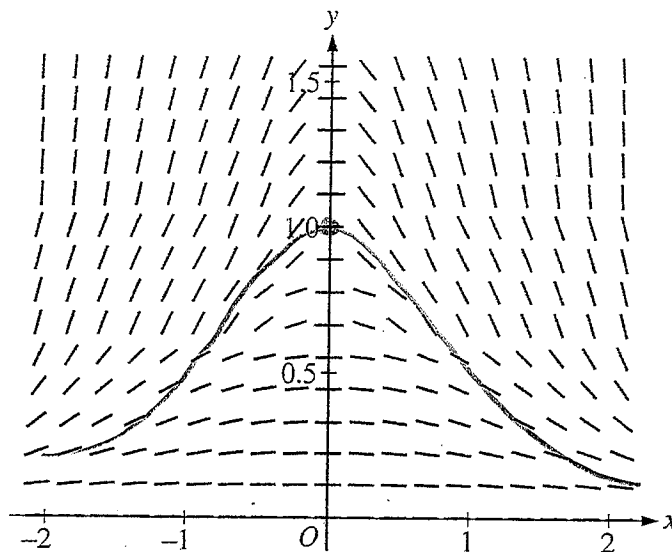
$$\int (x^6 + 2x^3 + 1) dx$$

$$\frac{x^7}{7} + \frac{x^4}{2} + x + C$$

14. $\lim_{h \rightarrow 0} \frac{e^{(2+h)} - e^2}{h} = \frac{0}{0}$ $\lim_{h \rightarrow 0} \frac{e^{2+h}}{1} = e^2$

← Talk about L'Hopital's Rule.

- Δ (A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$



15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

E

(A) $y = \cos x$

(B) $y = 1 - x^2$

(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \frac{1}{1 + x^2}$ ← $(1 + x^2)^{-1}$

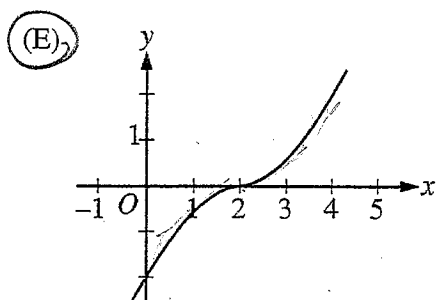
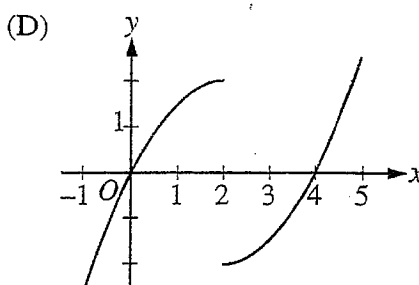
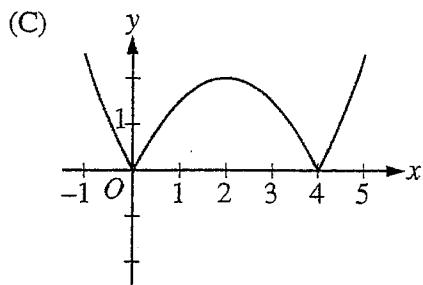
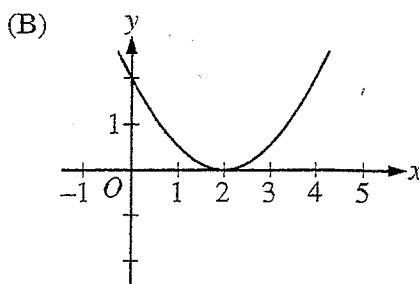
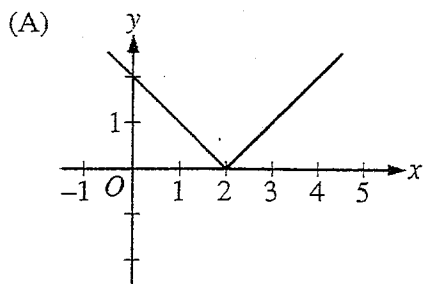
$y' = -(1 + x^2)^{-2}$

$y' = \frac{-1}{(1 + x^2)^2}$

x	y'
0	-1

16. If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

E



$$x(1-2x) \Rightarrow \text{vertex at } x=0, x=1/2$$

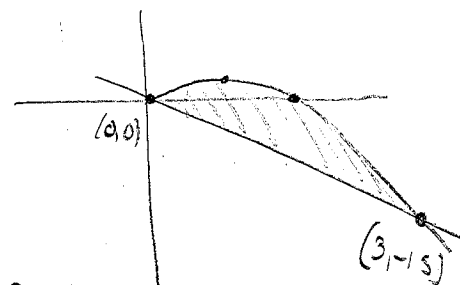
17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$?

- D (A) $\frac{7}{3}$ (B) $\frac{16}{3}$ (C) $\frac{20}{3}$ (D) 9 (E) 36

$$A = \int_0^3 (x - 2x^2) - (-5x) dx$$

$$= \int_0^3 (6x - 2x^2) dx$$

$$= \left[3x^2 - \frac{2}{3}x^3 \right]_0^3 = 9$$



D 18. For the function f , $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

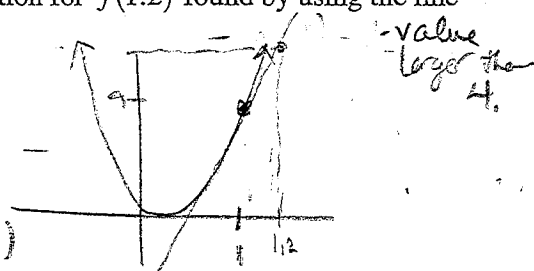
- (A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

$$f'(1) = 2(1) + 1 = 3 \leftarrow \text{slope of tan line thru } (1, 4)$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3(1.2 - 1)$$

$$y = 3.6 - 3 + 4 = 4.6$$



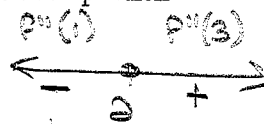
19. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

- A (A) $x > 2$
 (B) $x < 2$
 (C) $0 < x < 4$
 (D) $x < 0$ or $x > 4$ only
 (E) $x > 6$ only

$$f'(x) = 3x^2 - 12x$$

$$f''(x) = 6x - 12 = 0$$

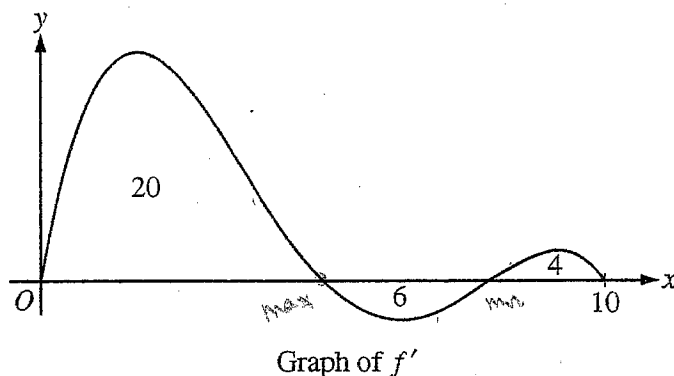
$$x = 2$$



20. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x) dx =$
- (A) $-\frac{14}{3}$ (B) -2 (C) 2 (D) 4 (E) $\frac{14}{3}$

$$\int_1^3 g'(x) dx = [x^2 - 3x + 4]_1^3$$

$$= 4 - (2) = 2$$



21. The graph of f' , the derivative of the function f , is shown above for $0 \leq x \leq 10$. The areas of the regions between the graph of f' and the x -axis are 20, 6, and 4, respectively. If $f(0) = 2$, what is the maximum value of f on the closed interval $0 \leq x \leq 10$?

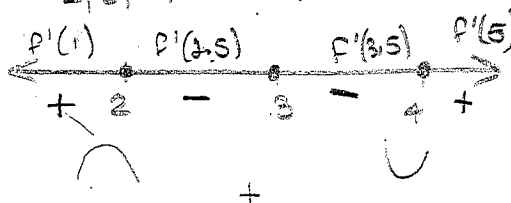
- (A) 16 (B) 20 (C) 22 (D) 30 (E) 32

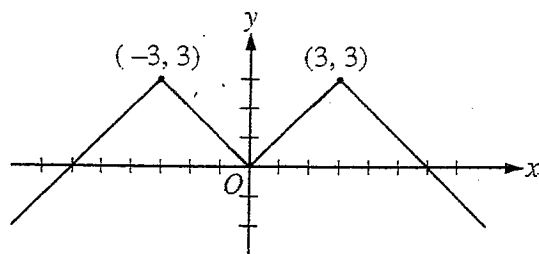
$$2 + 20$$

22. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

- A (I) I. A relative maximum at $x = 2$
 II. A relative minimum at $x = 3$
 III. A relative maximum at $x = 4$
 (A) I only
 (B) III only
 (C) I and III only
 (D) II and III only
 (E) I, II, and III

$$x = 2, 3, 4 \leftarrow \text{C.V.}$$





23. The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

B

(A) $\lim_{x \rightarrow 0} (f(x) - f(0)) = 0$ (T)

(B) $\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = 0$

(C) $\lim_{x \rightarrow 0} \frac{f(x) - f(-x)}{2x} = 0$ (T)

(D) $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = 1$ (T)

(E) $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$ does not exist. (T)

Even function so $f(x) = f(-x)$
all form of derivative.

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

D

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

25. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

A

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

$$\frac{dA}{dt} = 2 \cdot \frac{dC}{dt}$$

$$A = \pi r^2$$

$$C = 2\pi r$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

$$2\pi r \frac{dr}{dt} = 2 \cdot (2\pi \frac{dr}{dt})$$

$$2\pi r = 4\pi$$

$$r = 2$$

$$\frac{dy}{dx} = \frac{3 - 2xy}{x^2 - 3y^2} = \frac{3 - 2(-1)(2)}{(-1)^2 - 3(2^2)} = \frac{7}{-11}$$

26. For $x > 0$, f is a function such that $f'(x) = \frac{\ln x}{x}$ and $f''(x) = \frac{1 - \ln x}{x^2}$. Which of the following is true?

C

(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$.

(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$.

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$.

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$.

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.

$$\begin{array}{cc} f'(1/2) & f'(e) \\ - & + \end{array}$$

$$\frac{1 - \ln(2e)}{4e^2}$$

$$\begin{array}{cc} f''(1) & f''(2e) \\ + & - \end{array}$$

$$\frac{-(\ln 2 + \ln e)}{4e^2} = \frac{-\ln 2}{4e^2}$$

27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t} \, dt$, then $f'(2) =$

E

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

$$2\sqrt{4x^2 - 2x}$$

$$2\sqrt{4(2^2) - 2(2)}$$

$$2\sqrt{16 - 4}$$

$$2\sqrt{12}$$

28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} = \frac{5}{\sqrt{1-(5x)^2}}$

E

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

76. A particle moves along the x -axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t+2)$. What is the acceleration of the particle at time $t = 6$?

- C (A) 1.500 (B) 20.453 (C) 29.453 (D) 74.860 (E) 133.417

In calc $y = x^2 \ln(x+2)$

$\boxed{2^{nd}}$ calc 6: dy/dx $x = 6$ \boxed{enter}

77. If $\int_0^3 f(x) dx = 6$ and $\int_3^5 f(x) dx = 4$, then $\int_0^5 (3 + 2f(x)) dx = 3x \Big|_0^5 + 2(6 + 4)$

- D (A) 10 (B) 20 (C) 23 (D) 35 (E) 50 $15 + 20$

78. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

E

- (A) The change in temperature during the first day
 (B) The change in temperature during the 24th hour
 (C) The average rate at which the temperature changed during the 24th hour
 (D) The rate at which the temperature is changing during the first day
 (E) The rate at which the temperature is changing at the end of the 24th hour

$H = \text{temp}$

$H' = \text{change in temp}$
 at time = 24

79. A spherical tank contains 81.637 gallons of water at time $t = 0$ minutes. For the next 6 minutes, water flows out of the tank at the rate of $9 \sin(\sqrt{t+1})$ gallons per minute. How many gallons of water are in the tank at the end of the 6 minutes?

A

- (A) 36.606 (B) 45.031 (C) 68.858 (D) 77.355 (E) 126.668

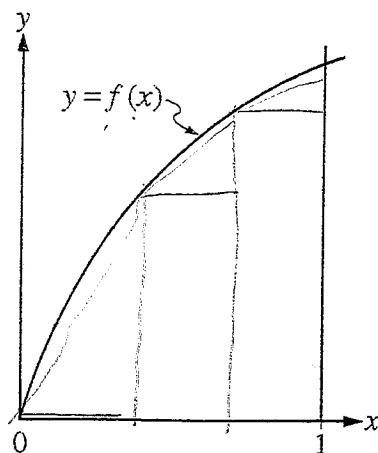
$V'(t) = 9 \sin(\sqrt{t+1})$

$\int_0^6 9 \sin(\sqrt{t+1}) dt$
 $= 45.031019$



$V = 81.637$
 when $t = 0$

$81.637 \text{ gal} - 45.031 \text{ gal}$



80. A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x) dx$, each using the same number of subintervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x) dx$?

I. Left sum \leftarrow under

II. Right sum

III. Trapezoidal sum \leftarrow under

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

81. The first derivative of the function f is given by $f'(x) = x - 4e^{-\sin(2x)}$. How many points of inflection does the graph of f have on the interval $0 < x < 2\pi$?

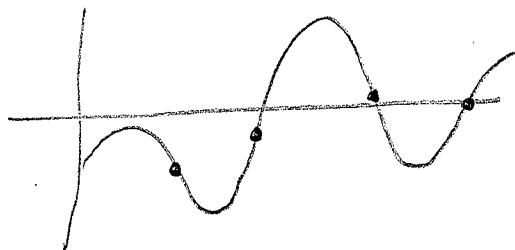
(A) Three

(B) Four

(C) Five

(D) Six

(E) Seven



82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

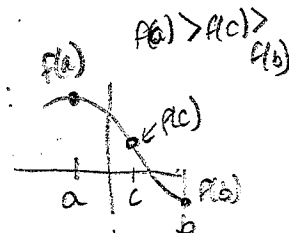
(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$. \leftarrow means there must be a maximum on closed interval.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$. \leftarrow no - abs. value graph.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.



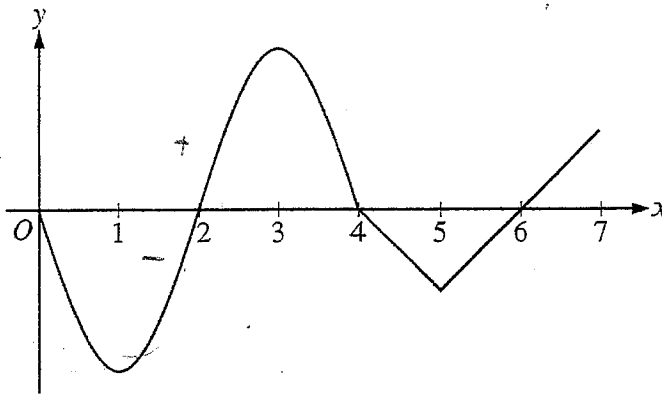
x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

83. The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \leq x \leq 5$. Which of the following could be the value of $f'(3)$?

D (A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5 $f'(3) = \frac{f(3.1) - f(2.8)}{3.1 - 2.8}$

$$= \frac{48.05 - 39.2}{3.1 - 2.8} = 29.5$$

29 would make it f' decreasing

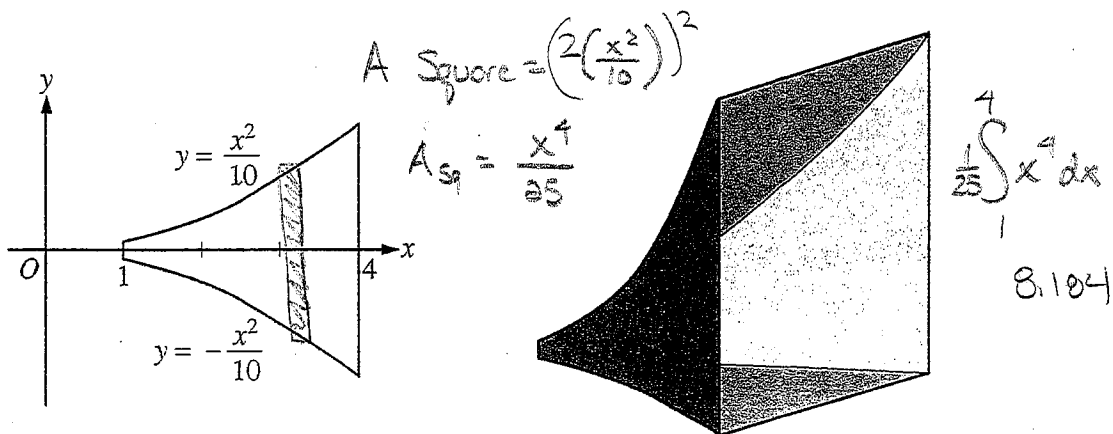


Graph of f'

84. The graph of f' , the derivative of the function f , is shown above. On which of the following intervals is f decreasing?

Decreasing where f' is negative.

- (A) $[2, 4]$ only
(B) $[3, 5]$ only
(C) $[0, 1]$ and $[3, 5]$
(D) $[2, 4]$ and $[6, 7]$
(E) $[0, 2]$ and $[4, 6]$



85. The base of a loudspeaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \leq x \leq 4$, as shown in the figure above. For this loudspeaker, the cross sections perpendicular to the x -axis are squares. What is the volume of the loudspeaker, in cubic units?

- (A) 2.046 (B) 4.092 (C) 4.200 (D) 8.184 (E) 25.711

x	3	4	5	6	7
$f(x)$	20	17	12	16	20

86. The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

I. The minimum value of f on $[3, 7]$ is 12. \leftarrow Could have a value of $(5.5, 10)$

II. There exists c , for $3 < c < 7$, such that $f'(c) = 0$. \leftarrow Rolle's Thm $f(3) = f(7) \leftarrow$ True.

III. $f'(x) > 0$ for $5 < x < 7$. f is cont + diff. So $f'(c) = 0$

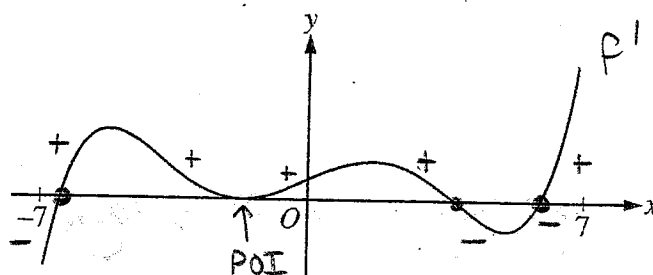
(A) I only

(B) II only

(C) III only

(D) I and III only

(E) I, II, and III



Graph of f'

2 min.
1 max

87. The figure above shows the graph of f' , the derivative of the function f , on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One

(B) Two

(C) Three

(D) Four

(E) Five

88. The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1+5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \leq t \leq 4$, what is the average rate of water flow, in gallons per minute?

(A) 8.458

(B) 13.395

(C) 14.691

(D) 18.916

(E) 35.833

$$\frac{1}{4-0} \int_0^4 2\sqrt{1+5t^3} dt = 14.691$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-2	-3	4

89. The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If

$$h(x) = (2f(x) + 3)(1 + g(x)), \text{ then } h'(1) =$$

(A) -28

(B) -16

(C) 40

(D) 44

(E) 47

$$\begin{aligned} h'(x) &= (2f(x) + 3)(g'(x)) + (1 + g(x))(2f'(x)) \\ h'(1) &= (2(3) + 3)(4) + (1 + (-3))(-2) \\ h'(1) &= 9(4) + (-2)(-4) \\ &= 36 + 8 \end{aligned}$$

← inverses.

90. The functions f and g are differentiable, and $f(g(x)) = x$ for all x . If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

E

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

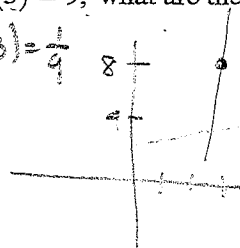
(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

$f(3, 8)$

$g'(3) = \frac{1}{9}$

$g(8, 3)$



91. A particle moves along the x -axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 5te^{-t} - 1$. At $t = 0$, the particle is at position $x = 1$. What is the total distance traveled by the particle from $t = 0$ to $t = 4$?

D

(A) 0.366

(B) 0.542

(C) 1.542

(D) 1.821

(E) 2.821

$\int_0^4 |v(t)| dt = 1.821$

92. Let f be the function with first derivative defined by $f'(x) = \sin(x^3)$ for $0 \leq x \leq 2$. At what value of x does f attain its maximum value on the closed interval $0 \leq x \leq 2$?

C

(A) 0

(B) 1.162

(C) 1.465

(D) 1.845

(E) 2

$\sin(x^3) = 0$

